ASSOCIATIONS BETWEEN CONFIDENCE, PERSISTENCE, AND OPTIMISM: ILLUMINATING OPTIMISTIC PROBLEMSOLVING ACTIVITY

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As part of video-stimulated post-lesson student interview studies of problem solving activity, Williams (2005, 2011) found successful problem solvers were optimistic. This study interrogates data from three students from a broader study: the role of optimism in collaborative problem solving and how to build it. The research question that focuses this study is “What associations exist between confidence, persistence, and optimism?” The question is explored both theoretically and empirically for the purpose of increasing understanding of the nature of optimistic problem solving activity by linking it to more familiar constructs. Students were progressively selected to further the interrogatory process. It was found that confidence and persistence did not necessarily occur together, and when they did, activity associated with these two characteristics was not sufficient for optimistic problem solving activity to occur. Perseverance was also required.

Key Words: confidence, persistence, optimism, optimistic problem solving activity, perseverance

INTRODUCTION

For over thirty years, mathematics education researchers have recognized that problem solving activity can ‘deepen’ mathematical understandings (Skemp, 1976; Cobb, Wood, Yackel, & McNeal, 1992; Williams, 2005). But, many students do not engage in such activity and may actively fight against it by demanding to be ‘told’ (Anthony, 1996). The Australian Mathematics Curriculum (see <http://www.australiancurriculum.edu.au/Mathematics/Rationale>) and the Melbourne Declaration of Educational Goals for Young Australians <http://www.mceecdya.edu.au/mceecdya/melbourne_declaration,25979.html> require the development of creative, innovative and resourceful problem solvers. More needs to be known about how to build these ‘characteristics’ where the term ‘characteristics’ is used in relation to constructs that describe problem solvers. My research (Williams, 2005) identified ‘optimism’ / ‘resilience’ (Seligman, 1995), as a characteristic of creative problem solvers. Given these links found between optimism and successful problem solving activity, it is thus important to develop a language that connects optimism with more familiar constructs to increase understanding of the meaning of optimistic problem solving activity and how it is enacted. The question explored is: “What theoretical and empirical links can be found between ‘confidence’, ‘persistence’ and ‘optimism’?
THEORETICALLY FRAMING THIS STUDY

Optimism is an orientation to failures and successes (Seligman, 1995). Optimistic children perceive failure as ‘temporary’ (able to be overcome), ‘specific’ (to the situation at hand), and ‘external’ (can be associated with factors beyond their control). They perceive successes as personal (achieved through their own effort), permanent (able to be achieved again), and pervasive (internalized as characteristics of self: ‘I did this, I am good at this’).

With regard to optimistic activity specific to mathematical problem-solving successes (‘optimistic problem-solving activity’), ‘failure’ is taken to be ‘not knowing’, and ‘success’ as ‘finding out’. An optimistic problem solver perceives ‘not knowing’ as temporary [Failure as Temporary] and able to be overcome through personal effort [Success as Personal] associated with looking into the situation of failure, and differentiating between what can be changed [Failure as Specific] and what is outside their control [Failure as External], to help decide what to vary to increase their likelihood of success [Failure as Specific]. They perceive they can achieve such successes again [Success as Permanent] because they internalize their successes as a characteristic of self [Success as Pervasive].

‘Persistence’ is taken to mean “how much students keep trying to work out an answer or to understand a problem even when that problem is difficult or is challenging … ‘If I can’t understand my schoolwork at first, I keep going over it until I understand it’” (Martin, 2003, p. 46). Martin’s ‘persistence’ construct contains two elements: ‘keep trying and trying’ and ‘until I understand’. In relation to optimism, these elements are consistent with perceiving Success as Personal, and Failure as Temporary (see Figure 1).

‘Confidence’ has been described as the “degree to which a person feels certain of her or his ability to learn and perform well in mathematics” (Hart, 1999, p. 243), and as relating to a personal characteristic: “… one's ability to learn and to perform well on mathematical tasks” (Fenemma & Sherman, 1976, p. 326). The two elements, certainty in ‘perform[ing] well’ and in ‘one’s own abilities’ to are consistent with the optimistic perceptions Success as Permanent and Success as Pervasive respectively (see Figure 1).

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<tr>
<th>NON-OPTIMISTIC PERCEPTIONS</th>
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Figure 1. Optimistic and non-optimistic perceptions displayed by dimension (see Left Hand Side) showing their theoretical relationships to persistence and confidence (see inside ovals).
Figure 1 displays optimistic perceptions and perceptions associated with lack of optimism along the three dimensions of optimism: Pervasive—Specific, Temporary—Permanent, Personal—External. Confidence and persistence have each been theoretically defined by two optimistic perceptions (see ovals in Figure 1). This study interrogates empirical evidence to find associations between optimism, confidence, and persistence by asking: do persistence and confidence always occur together? If not, what combinations can be found?

RESEARCH DESIGN

The three students selected to study included one whose characteristics changed over time. Their activity was expected to provide empirical evidence of associations between confidence, persistence, and optimism due to differences in characteristics they displayed.

The Students

These students attended either a Catholic, or Government, elementary school in Australia.

Lenny was an average performer on class tests. He participated in the study from Grade 4-6. Initially, he tended to ‘give up’ easily, but later in the study he reported that he now spent considerable time thinking about mathematical ideas during problem solving.

Sam participated in this study in Grade 6. In his class, he was the highest performing student on tests. He appeared confident in his mathematical abilities, but did not explore new ideas during problem solving tasks, and did not ‘deepen’ his rule-based understandings.

Bethany was a high performing student on class tests who participated in this study from Grade 4-6. She focused intently on the problem solving activities, and encouraged others to do so too. She appeared persistent but not confident about what she could achieve.

Tasks and Pedagogy

Students undertook three complex problem-solving tasks each year (six eighty-minute sessions) for one to three years. Tasks are accessible through a variety of representations and levels of mathematical sophistication to give students opportunities to discover and explore complexities just beyond their present understandings. Brief descriptions of aspects of the problem solving tasks relevant to the student data reported are now provided. More information about student responses during these tasks can be found in the papers cited.

Task 1. Shaping Rectangles Task (Williams, 2010): Using all 14 tiles (each time), make as many different flat ‘filled’ rectangles as you can (using top surface of tiles lying flat on table). Repeat using 12, then 16 tiles. Have you found all possibilities? Make an argument that justifies that you have them all. Can more rectangles be made when you have more tiles? Why or why not? What is the number of tiles between 16 and 45 that your group has found to make the most rectangles? Explain the thinking your group did to find this.

Task 2. The Fours Task (Williams, 2008): In this task students were required to make each of the whole numbers from one to twenty inclusive using four of the digit four and as many of the following operations and symbols as necessary:

\[ + - \times \div \sqrt (\ C) ^2 \]
The problem was initially attempted individually with pen and paper (three minutes), then ideas were shared in groups, and then groups worked on the problem. Each group had a set of 16 tiles (each number and symbol) that they could use to support these discussions. Groups reported to the class at regular intervals using a set of transparent tiles on an overhead projector. This enabled students to communicate using a combination of language and visual images (Ericsson & Simons, 1980), and also provided opportunity for reporters to shift symbols and numbers around to communicate ideas developed. During the first reporting session, groups could focus on any of the following:

- Two calculations that generated different numbers
- A pattern or system found
- Something that was causing trouble that another group might help with
- A ‘big picture idea’ that helped generate numbers faster, or
- Anything else they thought could be useful to other groups.

**Task 3. The Blue Smartie Promise (Williams, 2011):** The overarching task was to design an advertising slogan to market Smarties (colored candy) by constructing a Blue Smartie Promise to attract blue Smarties lovers to buy. Attention was drawn to broken promises being ‘bad’ for the Smartie Company. Each group predicted the number of blue Smarties in their small-unopened box of Smarties, and gave reasons for their prediction (first report). Then they opened the box, counted the blue Smarties, and discussed their findings: how they differ from their predictions, and their thoughts on why this might have happened (second report). They added their tally mark to the data on the board. The procedure was repeated with each student opened a box after predicting, then adding their tally mark to board (See Figure 2). Groups analyzed the data to decide on their promise, and then reported their decision to the class. The feasibility of keeping each promise was discussed.

The researcher (author) as teacher (RT) and the classroom teacher (T) team-taught with the RT as the primary implementer of the tasks. Students worked in small groups (3-4 students) and gave brief reports to the class at 5-10 minute intervals. The RT and T did not affirm pathways taken nor ideas presented but rather asked questions to stimulate further thinking. For more information about the teaching and learning approach, see Williams (2009).

**Data Collection Techniques**

Four video cameras captured the activity of each group during their problem solving sessions. This included capture of group interactions, and group reports to the class. After each session, the work from each group, and any artifacts produced were collected and used as extra stimuli in the four individual post-lesson video-stimulated student interviews.

In their interviews, students had simultaneous access to video of their group and the reporting sessions. They identified and discussed parts of the lesson that were important to them and reconstructed some of their thinking and feelings in the lesson. Video-stimulated interviews increase the validity of student reconstructive reports through focus on memory traces related to specific activity that occurred (Ericsson & Simon, 1980).
Students also answered questions designed specifically to provide data about their optimistic or non-optimistic perceptions through analysis of their descriptions and decision-making processes. For example: “How do you think you are going in maths, and how do you decide?” and “How do you learn something like that [mathematics associated with the problem solving task]?” tended to elicit information about whether students perceived Success as Personal or External. Where they primarily evaluated their performance themselves and perceived learning as predominantly associated with personal effort, and the creating of new ideas, they displayed indicators of Success as Personal. Alternatively, where they relied primarily on external judgments of their performance and described the way they learnt as through attention to external sources, they displayed indicators of Success as External. These questions sometimes elicited information about how students perceive their performances in terms of characteristics of self. For example: “I am really dumb at maths, I am never going to be able to do it” [Failure as Pervasive], and “I can do these, I am really clever” [Success as Pervasive]. Indicators of optimism or lack thereof were also sometimes displayed when students discussed how others might consider them when they ‘got something wrong’ (e.g., “they might all think I am an idiot”) [Failure as Pervasive], or “they would know that calculation is tricky” [Failure as Specific]. Questions like “Can you tell me what you were thinking about there? And, how did you work that out?” can elicit data about how a student altered variables to increase the likelihood of success [Failure as Specific]. Optimistic or non-optimistic indicators are not always evident in responses to a particular question, and the probes following each question depend upon the student’s response. Thus indicators of optimism are found in different parts of the interview transcript for different students.

RESULTS

Results are reported in the order in which the interrogatory process was undertaken. The transcription protocol used is as follows:

‘…’ omitted transcript that does not alter the meaning; ‘(…)’ inaudible; ‘/’ cut across the talk of another;
‘)’ link to the talk of another; ‘//’ simultaneous talk; ‘[text]’ researcher comment to clarify meaning; ‘-’ change in direction of talk; ‘(pause)’ pause break in talk.

Lack of Persistence

When Lenny participated in the initial problem-solving task in Grade 4 (Shaping Rectangles), he ‘gave up’ part way through the task. Where previous knowledge could easily be applied, Lenny participated. For example, when his group worked on the 12-tile case in, Lenny ‘saw’ that the 3 × 4 rectangle with the two tile remainder (shown by another group during trial and error on 14-tile case) showed this rectangle contained 12 tiles (14-2=12 so the 3x4 rectangle must contain 12 tiles). He excitedly contributed this idea to the group report for the 12-tile case. During the final task part, because his group had not recognized the relevance of factors, they used trial and error. Lenny stopped participating and started distracting others by poking them playfully at this stage.

Lenny’s later reflections about this activity confirmed he had given up: “Instead of just going ‘I don’t know’ [like I did earlier], I [now] sit there and I really really think about it-
…”. Lenny’s statement ‘I don’t know’ indicated he perceived ‘not knowing’ was permanent and his change to off task activity indicated he did not perceive personal effort would help.

Lenny’s activity in Grade 4 provides empirical evidence that a student may be neither confident of doing something unfamiliar, nor persistent in trying to do so.

**Persistent But Not Confident**

Bethany listened carefully to what was expected, and to group members’ ideas, and asked them for explanations when she did not understand. This activity was identified in her group problem solving at the start of, and in the last year of, the study. In one of her interviews in Grade 4, she described how she learnt in her usual maths classes: “[I]f I don’t really get it I really really try and listen- and sometimes I try and make ways that … [it] sort of makes sense and use different numbers in the same ways as he [T] was.” Bethany perceived ‘not knowing’ as temporary, and able to be overcome through personal effort exerted in ‘really really try[ing] … listening’: she was persistent.

She described liking group work because ‘if you get an answer wrong … the whole group took the fall’ showing she perceived failures were taken on as attributes of self [Failure as Pervasive] but diluted if the whole group was responsible for the failure. Bethany was not confident because she took on her failures rather than her successes as attributes of self. Bethany’s activity provides empirical evidence that a student can be persistent but not confident. The next question is: Can a student be confident but not persistent?

**Confident But Not Persistent**

When Sam was asked “What’s learning something for you?” he responded “… gaining, knowledge (pause) sometimes I read … somebody teaches … I go on the Internet”. Sam perceived ‘finding out’ (success) was achieved by consulting expert sources [Success as External], rather than by developing ideas himself [Success as Personal]. He perceived he was able to learn fast (find out new things) when ‘told’ [Success as Permanent] because he was ‘good at maths’ as demonstrated by his test scores [Success as Pervasive]. Although Sam displayed confidence [Success as Permanent, Success as Pervasive] he did not display persistence For example, in Task 2, The Fours Task, Sam stopped working after one minute (and covered his work) even though he had only generated eight of the twenty required calculations (See below). Sam only generated answers by using operations separately (e.g., $4/4 + 4 + 4 = 9$) rather than in a composite way (e.g., $(4+4/4) \times .4 = 2$). He did not ‘step outside’ his present understandings to explore unfamiliar ways to work with known operations and symbols. Although Sam’s number fact recall was fast (demonstrated by him generating eight calculations faster than others who eventually generated more), two of his calculations were incorrect, and he displayed less sustained use of pattern (with two pairs only), and showed less evidence of experimentation.

\[
\begin{align*}
4/4 + 4 + 4 &= 9; \\
4/4 + 4 - 4 &= 1; \\
\sqrt{4} + 4 + 4 - 4 &= 6; \\
\sqrt{4} \times 4 + 4 + 4 &= 20; \\
4 + 4 + 4 - 4 &= 8; \\
4/4 + \sqrt{4} + 4 &= 7; \\
4/4 + 4 \times 4 &= 17; \\
4 \times 4 + 4 - 4 &= 5
\end{align*}
\]
During group sharing time, Sam did not share. He took and copied the work of others and worked alone to produce similar calculations. Sam did not enact creating new ideas through personal effort but rather learning from external sources [Success as External]. Sam’s activity provides empirical evidence that confident students are not always persistent. Sam’s and Bethany’s activity raise questions about whether confidence and persistence can co-exist, and whether all students who possess both characteristics are also optimistic.

**Persistent and Confident**

As shown by the quote from Lenny reported earlier, by the end of Grade 5 Lenny had shifted from ‘giving up’ [Failure as Permanent], to knowing if he exerted a great deal of personal effort by ‘really really think[ing] about it’ [Success as Personal] he would be able to ‘find out’ (success) more [Failure as Temporary]. Lenny had developed persistence.

When asked how this happened, Lenny replied reflectively: “I don’t really know (pause) … it just made me think more. It was probably actually doing the tasks …”. Lenny displayed Success as Pervasive by recognizing a new attribute of self (persistence) when he recognized he now made an effort to really really think. Implicit in his comment ‘actually doing the tasks’ is an acknowledgement that he now perceives he can problemsolve because he now thinks more (Success as Permanent). Lenny has now displayed evidence of confidence. Lenny did enact optimistic problem solving activity as demonstrated below.

Figure 2 shows the data generated on the board during the Blue Smartie Promise Task as students counted the number of blue Smarties in boxes (at the end of Lenny’s second year in the study). Figure 2 is interpreted as follows: the rectangle with a six in it represents boxes containing six blue Smarties and the tally of five beside it shows there were five such boxes.

![Diagram on board: tallies of numbers of blue Smarties in boxes](image)

The following interview excerpt shows Lenny’s surprise on inspecting the data, and the spontaneous challenge (just beyond his present understanding) he set himself as a result: find the average. It also shows how he thought hard about what was the matter with his calculation of the average as approximately two, and recognized what needed to be changed and why. Line numbers show where the transcript is not consecutive.
1. Lenny … [another] group had (pause) one [blue Smartie] (pause) … I found that really really surprising (pause) … even the four [blue Smarties] (pause) because that is half (pause) what I thought it would be (pause)

2. Interviewer And why do you think that happens?

3. Lenny It is probably (pause) when they were putting them in the boxes it is just (pause) random

4. Interviewer … Had you thought that out at that time? Or (pause)?

5. Lenny [intense] Yeah I was trying to think (pause) what the (pause) average was

6. Lenny And I think I did it wrong … I counted the fifteen as one

8. Lenny … I just counted all the ones that had (pause) … numbers next to them …

Lenny was surprised about the data on the whiteboard [Line 1]. He had not expected boxes with only one or four blue Smarties [Line 1]. Four blue Smarties in a box was half his own prediction [Line 1]. He spontaneously formulated a question to explore this complexity he had discovered: What is the average? [Line 5]. He knew there was something the matter with the size of the average he generated, and began to look into what he had done to identify what needed to change [Line 6].

24. Interviewer How did you know there was something the matter with what you did?

25. Lenny Because I knew the (pause) … if there was eight (pause) six and five each (pause) mmore of them are over five so how is it [the average I calculated is] under two?

27. Lenny And I knew probably it would be around five six because the one [one blue Smartie box] would bring it down a fair bit …

35. Lenny … But I-I like di- I just went one (pause) two and stuff but I didn’t count like (pause) … yeah I didn’t count all of them as 15 (pause) I just counted them as one each [getting a total of nine]

Lenny knew his calculation of average did not match his inspection of the data [Lines 25, 27], he looked into the situation to see what needed to be changed to increase his likelihood of success: “… I didn’t count all of them as 15 I just counted them as one …” [Line 35] [Failure as Specific]. Lenny recognized that the box of 15 blue Smarties should have a count of fifteen not the ‘one’ that resulted from him counting each box with tallies beside it as one [Line 35]. Although the class’ work on the task had finished before Lenny’s interview, he continued to puzzle intently about what was the matter with his calculation of average as illustrated by the emphases in his responses. Lenny enacted persistence.

Lenny was enacting optimistic problem solving activity and no non-optimistic activity. His activity provides empirical evidence that a) persistence and confidence can occur simultaneously; b) these characteristics can develop over time and successes achieved from ‘really thinking’ are a contributing factor to this change; and c) the perception of Failure as Specific is crucial to successful problem solving activity. This finding raises questions about whether students who are both confident and persistent are always optimistic (like Lenny).
DISCUSSION AND CONCLUSIONS

Theoretical links made between persistence, confidence, and optimism were able to be explored empirically using this research design to identify indicators and enactments of these characteristics. The links made extend the language available to support the processes of identifying, and discussing, optimistic problem solving activity. Overlaps between ‘confidence’, ‘persistence’, and ‘optimism’ have been identified (See Figure 3): students were identified who possessed none (Lenny initially), one (Sam, confidence; Bethany persistence), or both (Lenny in second year) of the characteristics ‘confidence’ and persistence’. Data from the students selected for study did not show whether a student could possess the characteristics represented by the space ‘X’ in Figure 3: persistent and confident but not optimistic. Further interrogating of the broader data set could identify such students. Not finding such students would add strength to the possibility of their non-existence.

![Figure 3. Relationships found between persistence, confidence, and optimism](image)

The optimistic attribute Failure as Specific enacted by Lenny and the persistent activity enacted by Bethany draw attention to a distinction between persistence and perseverance: “Perseverance means that you invest your time to accomplish something. Persistence means that you stay in a particular mode or situation until you achieve your goal” (Fowler, 2004, p. 4). Distinguishing between persistence and perseverance illuminates the nature of perseverant problem solving activity: “here’s another insight that some may overlook. Part of the skill of the power of perseverance is to make those adjustments as you persist” (Conroy, 1999, p. 30). Adjusting whilst persisting enacts Failure as Specific. The productiveness of adjustments relies on the ability to distinguish between what is and is not within the problem solvers’ control [Failure as External]. Thus, ‘perseverance’ is defined by the other two optimistic perceptions: Failure as Specific and Failure as External, and optimism is the simultaneous possession of ‘confidence’, ‘persistence’, and ‘perseverance’.

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REFERENCES


