ANALYSIS OF ALGEBRAIC ERRORS IN APPLIED CALCULUS PROBLEM SOLVING
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The purpose of this paper was to assess algebraic prerequisites skills as incorporated into the Applied Calculus Optimization Problem (ACOP) solution. The difficulties that students encountered in applying algebraic prerequisites at the early stages of the ACOP solution were identified. The study analyzes errors related to variables and equations (i.e. algebraic symbol/transformation skills), and those associated with application of basic differentiation concepts into ACOP solution process. In general, they failed to integrate the basic algebraic competences required in ACOP solution procedure. In cases where integration occurred, students face structural and procedural setbacks that ultimately led to a weakening of the ACOP solution process.

Key words: optimization, variables, equations, transformation skills.

INTRODUCTION
Calculus is a branch of mathematics which has been developed to describe relationships between two or more things which can change continuously (Davis & Hersh, 1981). According to Young (1986) quoted by Ferrini-Mundy and Graham (1991), “calculus is our most important course . . . the future of our subjects depends upon improving it” (p. 627). Despite the importance of calculus, certain difficulties inhibit students from learning it, leading to unprecedented failure. As stated by Ferrini-Mundy and Graham, out of about 600,000 students that enrolled in calculus in four-year colleges and universities in United States of America, less than half finish with a grade of D or higher. This clearly shows that calculus was either poorly learned or taught or both.

Some of the calculus-learning inhibition factors can be particularly related to the characteristics of first-year calculus course (Burton, 1989). According to Burton, first-year calculus course relies on some specific mathematics skills that students are presumed to have mastered in high school. This includes the language and skills of Mathematics symbols, diagrams, equations and formulas (i.e. the fundamentals). Calculus taken during the first year in college clearly requires and depends on students’ actual mathematics skills that cut across algebra.

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A good window in which to study the problems associated with calculus learning was applied calculus optimization problems (ACOP). ACOP is the set of word problems which requires the application of algebra, geometry, and basic differentiation skills from calculus to compute, interpret and analyze larger and smaller values of a model on some interval, and determine where the largest or smallest value occurs. ACOP is a good piece of calculus to study because; ACOP incorporates the full range of mathematics language fundamentals including symbols, diagrams, equations, and formulas. In this respect, we can gain a full picture of how these fundamentals are applied in calculus through examination of ACOP. This characteristic makes ACOP extremely useful as an avenue to understand students’ broader calculus difficulties. I have decided to write a paper on this topic, which is an extract from my dissertation.

**LITERATURE REVIEW**

To some people algebra is a collection of symbols, rules and procedures, while to Mathematicians; it is much more than that. From the perspectives of Kieren (1992), algebra is conceived as a branch of Mathematics that deals with symbolizing and generalizing numerical relationships and mathematical structures, and with operation within those structures. From other perspectives, algebra is about identifying patterns and generalizing those patterns. Generalization involves seeing a pattern, expressing it clearly in verbal terms, and then using the symbols to express the pattern in general terms. Two fundamental concepts in algebra i.e. variables and equations are reviewed.

**Variables**

According to Sfard (1995), Mathematics historians seem united in the opinion that the 16th century French Mathematician François Viète was the first to replace numerical givens with symbols, variables. Students encountered the concept of a variable for the first time in introductory algebra or beginning algebra. This is followed by learning of algebraic terms and expressions, before finally getting into equations in college algebra, and general Mathematics. This arrangement is based on the fact that equations involve expressions and terms, but expressions and terms in turn contain variables. Understanding the concept of a variable is fundamental to the study of algebra which in turn forms the foundation of solution of different types of problems that are formulated into equations or set of equations.

A major area of research on variables was on students’ difficulty operating with variables as well as its misconceptions. Studies have suggested that very few students understand variables at highest conceptual level, White & Mitchelmore (1996), and Kieran (1989). Further studies, example Booth (1989) citing the work of White & Mitchelmore suggested that the required meaning of variable is often neglected in the teaching and learning of algebra. Many students only learn manipulation rules without reference to the meaning of the expressions being manipulated. It is a matter of some interest to find out whether students who aspire high and want to advance their mathematical thinking that may involves calculus and applied calculus optimization problem in particular have an adequate understanding of the concept of a variable.
Equations

Another important concept in algebra is equation, but the teaching and learning of algebra forms a significant component of the NCTM curriculum standards. It was emphasized in the curriculum that continued study of algebraic concepts such as [equations, variables] would help students represent situations that involve variable quantities with expressions, equations …, and use tables, graphs [and calculus] as tools to interpret expressions, equations, NCTM (1989). The success of any applied calculus optimization problem (ACOP) relies heavily on a good understanding of above concepts in algebra, i.e. equation and variable. To stress the importance of variables and equations, Rosnick & Clement (1980) cited by Usman (2008) opined that the fundamental concepts of variables and equations should not be treated lightly in high schools and colleges, nor should we assume that our students will develop the appropriate concepts by osmosis.

Generating equations to represent the relationships found in typical word problems is well known to be an area of difficulty for algebra students, Kieren (2007). According to Kieren, cited by Usman (2008), word problem situations not only continue to be used as a means for infusing algebraic objects with meaning but have also received increased emphasis in reform programs as vehicles for introducing students to algebra. Furthermore, she opined that, research in this area continues to provide evidence of students’ preferences for arithmetic reasoning and their difficulties with the use of equations to solve word problems (e.g. Cortés, 1998; Swafford & Langrall, 2000). Is this possible in the solution of ACOP? Usman quoted Stacey and MacGregor (1999) as cited by Kieren found that, at every stage of the process of solving problems by algebra; students were deflected from the algebraic path by reverting to thinking grounded in Arithmetic problem-solving methods. Other researchers have concentrated on the difficulties associated with two major ways (syntactic and semantic) of translating equation from verbal data, (Kirshner et al, 1991; and Laborde, 1990).

Another aspect that contributes to students’ difficulty in translating word problems into equations is language, (Kaput, 1987).

Purpose of the Study

The purpose of this study was to assess algebraic prerequisite skills as incorporated into the ACOP solution. Generally, the study’s main goal was:

Assess students’ prerequisite skills in setting up an ACOP problem.

Specific research questions designed to address the main goal of the study are further subdivided into two quantitative and qualitative components.

Specific research questions are:

1a) Are students able to label constructed diagrams using appropriate variables?

b) How do students label diagrams using appropriate variables?
2a) Are students able to associate geometric diagrams with appropriate algebraic equation(s)/formula(s)?

b) How do students associate geometric diagrams with appropriate algebraic equation(s)/formula(s)?

3a) Are students able to use symbol/transformation skills?

b) What are students’ strengths/weaknesses in symbol/transformation skills?

4a) Are students able to solve ACOP completely?

b) How do students solve ACOP completely?

The quantitative components of the research questions are (a) parts while the qualitative components are the (b) parts. The overall outcome from these research questions was a synthesized model of how the prerequisites fit together to constitute a solution of ACOP.

Methods

This study applies a mixed methods research design. The data collection procedures involves gathering both numeric information (e.g., on instruments) as well as text information (e.g., on interviews) so that the final database represents both quantitative and qualitative information. For the purpose of this study, the quantitative portion of the sequential explanatory mixed method research was used to assess students’ algebraic prerequisites and basic differentiation skills in ACOP solution. The second stage of the sequential explanatory strategy is qualitative, using clinical interviews (stimulated recall).

The two processes of the sequential strategy, quantitative methods followed by the qualitative methods, were used to detect errors and misconceptions with respect to labelling of given diagrams with appropriate variables, associating geometric diagrams with appropriate formulae and transformation/symbols skills. Overall, the research analyzed full ACOP solution to determine the algebraic points of breakdown.

Demographics of participating students

The target population was freshmen students taking calculus I in the Department of Mathematics, Louisiana State University, Baton Rouge. The course was taught in the Fall, Spring and Summer semesters every year. For the purpose of this study, data were collected using students enrolled in the calculus I course in Spring semester, 2008. The portion of calculus I which was used for the study was applied calculus optimization problems.

Students taking calculus I are split into twenty-six different sections. Each section contains not more than forty students. This represents a total enrolment of 1040 students. Six sections of calculus I were used for the study. This represents 156 students or approximately 15% of the total enrolment in calculus I. The six sections used for the quantitative portion of the study were selected using the convenience sampling technique.
Quantitative/Qualitative Data Analysis

Table 1: Descriptive Statistics of all Variables and Subjects.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Mode</th>
<th>Std. Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Maximum possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labeldiag*</td>
<td>6.34</td>
<td>7</td>
<td>1.5</td>
<td>1</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Assdiag*</td>
<td>4.7</td>
<td>6</td>
<td>1.41</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Mansym*</td>
<td>3.78</td>
<td>4</td>
<td>1.69</td>
<td>0</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Optpbm*</td>
<td>1.83</td>
<td>2</td>
<td>1.41</td>
<td>0</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>


Four variables used in the study were grouped into two, isolated and non-isolated tasks. Isolated cases are tasks that require students to solve directly. The non-isolated tasks are where algebraic knowledge comes to play in the solution processes of complete word problems. The isolated tasks are, Label a diagram, Associating a diagram with formula and Manipulating symbol skills. On the other hand, the non – isolated task was deriving full ACOP equation and its solution. Overall, base on the statistical information (see Table 1), it was inferred that majority of students in the sample have shown some strength of performance in the isolated tasks, with relative exception to Manipulating symbol skills. Further evidence of the high - quality performance was manifested in the lower variability among the isolated cases. They were able to develop understanding and follow procedures and make connections where necessary (in explicit and implicit cases) in order to successfully complete the tasks as required. Still within the isolated tasks, there are some that tend to follow the procedures mindlessly, without making connections where necessary.

On the other-hand, i.e. the non–isolated tasks, statistical evidences (mean scores and standard deviation) indicated that, generally, students’ performance was lower than that of the isolated tasks. Comparing the mean scores for the three non–isolated tasks to that of the isolated, it comes out they were by far sizeable. Similarly, the standard deviation (table 1) are also higher, pointing out that there are more variability among them than the isolated tasks. Majority of students were unable to explore, understand the nature of the mathematical concepts, processes and establish relationships as required by the tasks. Some found it hard to regulate their thoughts in connection to the tasks. The end product of all these was that they failed to incorporate algebraic competences into full ACOP solution processes.

As earlier stated above, the qualitative data generated are also categorized into two, and are, isolated and non-isolated tasks. The isolated tasks are those that require algebraic expertise (symbol manipulation skills), while the non-isolated tasks requires the incorporation of those competences stated above to derived required ACOP equation and its solution.
Generally, students have performed well in the isolated tasks, but few exhibit some algebraic deficiencies. On a similar note, lack of students’ mathematical language have hindered labelling task success, but the non-canonical task was relatively accomplished with majority of the students establishing a good relationship between the sides (longer and shorter horizontal and vertical) sides of given geometric figure.

The qualitative data from another isolated task indicate that students have experienced some hardship with algebraic processes (adding algebraic fractions and connections to non-algebraic situation (trigonometric fractions), negative/positive signs errors in computation (non-intentional or otherwise)); structural (recognition of quadratic solutions, identifying quadratic equation in trigonometric form); and connection (use of appropriate trigonometric identities in the solution process, and detecting some erroneous solution of theta and eliminating it from the solution set).

Lastly, the qualitative data up-and-coming from last research question i.e. four, indicates that students have structural troubles finding the derivative of negative polynomial, accompanied critical/stationary point calculation as well as visualization, drawing, labelling of full ACOP. It can be argued that they found it difficult to access appropriate knowledge (algebraic); use their previous understanding and make connections in an interwoven way towards the solution of the tasks. Furthermore, it was clearly reveal that some students can’t have control or regulate their cognitive processes during the solution procedures of the tasks.

Discussion and Conclusion

The focal point in this study was to measure students’ skills or their absence in the complete ACOP solution process. Evidence from the statistical output for the complete ACOP task showed a mean score of 1.83 out of 9 possible points, with a standard deviation of 1.41, and a mode of 2. The minimum point scored was 0, while the maximum was 6.

The whole sample performance on this task based on this output shows that 106 students (69.4%) scored between 0 to 2 points, while 49 students (31.6%) achieved a score between 3 and 6. Drawing conclusion based on these facts indicated that a significant portion of the sample was unable to solve a complete ACOP successfully. Why are these students in calculus I who have just been taught ACOP solving unable to solve typical ACOP problems? This is the context in which the intention of the conclusions was base.

Overall, an accumulation of structural and procedural errors help in worsening the solution processes of non-algebraic equations (trigonometric). Most common among operational breakdown is inability to find the sum of two trigonometric fractions as well as clearing the resultant fraction. Moreover, students find it hard to identify, recall, and use appropriate trigonometric identity, in addition to recognizing quadratic structures express in trigonometric terms in order to get its solution.

In conclusion, the analysis of both quantitative and qualitative results reveals that, generally, students have demonstrated good propensity in isolated tasks that require algebraic expertise
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(symbol manipulation skills), but was unable to incorporate those competences in some non-isolated situations (equation derivation and full optimization problems).
References


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