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SECONDARY TEACHER CANDIDATES' MATHEMATICAL KNOWLEDGE FOR TEACHING AS DEMONSTRATED IN THEIR PORTFOLIOS

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This study focused on prospective secondary school teachers' mathematical knowledge and their ability to demonstrate how their learning of mathematics from their university courses applies to the teaching of secondary school mathematics. Prospective teachers enrolled in Advanced Mathematics for High School Teaching were required to develop a mathematics portfolio demonstrating their understanding of advanced mathematical contents and processes. Based on analysis of the portfolios, the majority of high school teacher candidates have difficulty fully demonstrating their mathematical ability, especially their ability to make connections between different levels of mathematical content areas. This difficulty was particularly evident in discrete mathematics and probability and statistics. In addition, teacher candidates struggled over finding connections between content and process standards.
Mathematics knowledge, teacher candidates, secondary level, portfolios

BACKGROUND AND RESEARCH QUESTIONS

In his well known article, Shulman (1986) emphasized the importance of teachers' pedagogical content knowledge (PCK) to improve teaching and student learning. In recent years, Hill, Ball and colleagues have advanced Shulman's original thinking on PCK to mathematical knowledge for teaching (MKT), which includes common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT) (Hill, 2010; Hill & Ball, 2004; Hill, et al. 2007). In an effort to measure elementary and middle grade teachers' mathematical knowledge for teaching, Hill, Ball and colleagues developed a multiple choice survey instrument. Using this instrument they found that MKT significantly correlates with teacher characteristics likely to improve positive teaching practices in the classroom. In order to measure the content knowledge and PCK of middle-school teachers, Saderholm et al. (2010) developed the Diagnostic Teacher Assessment in Mathematics and Science (DTAMS). DTAMS consisted of both multiple choice and open response items. Saderholm and colleagues provided detailed information in support of the reliability and validity of this assessment. A sample of 1600 teachers completed DTAMS. Based on results from this sample, they found that, on average, teachers "knew approximately 50% of the content necessary to teach mathematics to their students" (p. 187).

Although significant advances have been made in measuring PCK and MKT of elementary and middle school teachers, research on measuring MKT of secondary school teachers or preservice teachers is scarce. In the United States, the assessment of teachers' content knowledge is emphasized by the National Council for Accreditation of Teacher Education (NCATE) and the National Council of Teachers of Mathematics (NCTM). NCATE (2008) states that teacher education programs at a minimum must prepare candidates who "have the content knowledge needed to teach students" and "have the pedagogical and professional knowledge needed to teach effectively" (p. 7). NCTM has developed specific content and process standards required for teachers to be able to teach. For example, one of its recent volumes has focused on the importance of reasoning and sense making in high school mathematics (NCTM, 2009). There are other volumes that have emphasized the importance of studying secondary mathematics from an advanced perspective (Sultan & Artzt, 2011; Usiskin, Peressini, Marchisotto, & Stanley, 2003).

To ensure that teacher candidates have necessary mathematical content and pedagogical content knowledge required for teaching, NCATE and NCTM have collaborated in the development of standards (NCATE/NCTM, 2003) used in the accreditation of mathematics teacher education programs in the United States. Currently, teacher candidates are required to demonstrate their content knowledge through Praxis II (10061), which is a standardized multiple choice test administered by the Educational Testing Service (ETS). Although Praxis II is believed to capture preservice teachers' broad knowledge of mathematical content, it does not capture preservice teachers' PCK or MKT. In addition, Praxis II does not directly assess a preservice teacher's ability to integrate content from different undergraduate mathematics courses or to demonstrate connections between undergraduate and secondary mathematics.

In order to address the NCATE/NCTM (2003) performance expectations, this study illustrates how faculty members from both a Mathematics and an Education Department at a university in the United States collaborated to develop a capstone course for prospective high school mathematics teacher candidates. The course *Advanced Mathematics for High School Teaching* requires students to complete a mathematics portfolio in which they demonstrate their understanding of some central mathematics concepts in calculus, discrete mathematics, geometry, linear and abstract algebra, as well as articulate relationships between these concepts and major topics in high school mathematics.

The purpose of this paper is to document how a portfolio assessment with an accompanying rubric was developed and implemented in an advanced mathematics course and how the portfolio data could be used to assess secondary school teacher candidates' knowledge of mathematical content and processes associated with MKT.

More specifically, this study answers the following questions:

1. How strong is potential teacher candidates' mathematical content knowledge and their ability to connect college mathematics to the teaching of secondary mathematics?
2. Is there a significant difference between their knowledge of mathematical content and processes?

METHODOLOGY

The Course and Portfolio

The primary data source in the project was student portfolios created in *Advanced Mathematics for High School Teaching*. Based on the analysis of existing course outlines in *Discrete Structures*, *Calculus I*, *II*, & *III*, *Abstract Algebra*, *Applied Linear Algebra*, *Geometry*, and *Applied Probability and Statistics*, assessment materials, and the NCATE/NCTM (2003) standards, a list of key topics and concepts was developed and then used to create a rubric to measure the quality of candidate portfolios required for the course. The portfolio required teacher candidates to demonstrate knowledge of mathematics content in seven areas: number and operations, different perspectives on algebra, geometries, calculus, discrete mathematics, statistics and probability, and measurement. The other six areas focused on mathematical processes: Problem solving, reasoning and proof, communications, connections, representation, and technology. The mathematical content involved a total of 45 indicators and the mathematical processes included a total of 19 indicators. For example, an indicator under the number and operations (a content standard) reads as, "Recognize matrices and vectors as systems that have some of the properties of the real number system." An indicator under the mathematical problem solving (a process standard) reads "Solve problems that arise in mathematics and those involving mathematics in other contexts." Generally the NCATE/NCTM content standards are more specific than the process standards.

Pilot Study

The course was first taught on a pilot basis with an enrollment of 22 potential teacher candidates. A portfolio rubric was developed to measure the candidates' ability to demonstrate their understanding of mathematical knowledge. The pilot rubric and its development process can be found in Koirala and Johnson (2008). Each teacher candidate's response to the portfolio task in the pilot study was independently rated by a mathematics educator and a mathematician. The data were analyzed to determine candidates' mathematical content knowledge and the interrater consistency between the two raters. The interrater consistency on the set of scores on 13 process and content standards for the two raters was moderately high, $r = 0.739$ ($p < 0.001$). However, based on the data collected, it was clear that the majority of the candidates were confused about the portfolio task and expectations. For example, a few candidates submitted more than 50 portfolio entries without making explicit connections between mathematical topics such as algebra and geometry. Also, most of the problems presented in many of the portfolios were not at a level above high school, hence did not get into advanced college level mathematics.

The portfolio assessment data from the pilot study were used to improve both the course content and assessment rubric. (The revised rubric is provided in Appendix A.) The rubric includes qualitative descriptors of prospective teachers' ability to demonstrate their mathematical understanding in terms of "Target," "Acceptable," "Unacceptable" and "Missing", which were scored from 3 to 0, respectively. The rubric focused on students'

advanced mathematical thinking, elegance, mathematical connections and correctness, and ability to connect advanced university mathematics to high school topics.

Revised Course and Portfolio

In the revised course, teacher candidates were required to submit six portfolio entries, one each in number systems, algebraic structures, shape and measurement, function, discrete mathematics, and data analysis and uncertainty. They could develop their entries based on sample exams, papers, projects, and homework problems that they had completed in their other advanced mathematics courses. They were also allowed to develop entries based on library and internet research. Teacher candidates were allowed to exhibit their understanding of process standards through one or more of these six entries. It was emphasized throughout the course that they were required to develop these entries very carefully to address their ability to make explicit connections to secondary mathematics topics addressed by the NCATE/NCTM content and process standards. Teacher candidates were given extensive scaffolding through a 30-page portfolio booklet, which consisted of detailed descriptions of how entries could be developed from selected problems. The booklet also included examples of quality entries and a detailed description of the rubric.

The portfolio assignment allowed prospective teachers to demonstrate their understanding of some central mathematics concepts in abstract and linear algebra, calculus, discrete mathematics, geometry, and probability and statistics. The course also required prospective teachers to make connections among these content areas and to major topics in high school mathematics.

The revised course has been taught five times over a period of five years. Each time the course was taught, the portfolios were independently scored by a mathematics faculty and a mathematics education faculty. Based on the full rubric, each teacher candidate received a total of 13 scores ranging from 0 to 3 from each rater. The scores were analyzed using various quantitative techniques. First, interrater consistency analysis and its statistical significance were computed to determine the consistency of scoring between the two raters. Second, the means and standard deviations of each score were computed. For example, the authors computed each candidate's average score in their ability to demonstrate their knowledge of geometry and apply it to high school mathematics. Finally, a two-tail *t*-test was conducted to determine if the mean scores between the content and process standards were statistically significant.

RESULTS

In this study, we are providing data from two years. In the first year of teaching the revised course to 14 students, a strong interrater consistency was noted as indicated by the Spearman rank correlation of 0.863 ($p < 0.001$) and a 67% matching of scores between the two raters. It was determined that 11 out of 14 candidates satisfactorily demonstrated their understanding of all standards, including six process standards (problem solving, reasoning and proof, communication, connections, representation, and technology) and seven content standards (number and operations, algebra, geometry, calculus, discrete mathematics, statistics and probability, and measurement). Their entries and commentaries in the portfolios were

satisfactory both in terms of their mathematical understanding and their ability to show how university and high school mathematics topics are deeply connected.

The candidates' mean score of all the standards was 2.187, slightly higher than the acceptable average score of 2.000. The candidates' highest mean score was 2.357 on mathematical communication and lowest mean score was 1.821 for probability and statistics. Their scores were also low in discrete mathematics, with an average of 2.000. It was found that two candidates had not taken a probability and statistics course and hence, had not submitted an entry on this topic because of their lack of knowledge in this area. The scores in discrete mathematics were low primarily because of inconsistency of coverage of topics in the course *Discrete Structures*. Many candidates were confused simply because they did not know what would be an acceptable entry in discrete mathematics.

Another interesting finding of this study is that candidates' scores in the content standards were lower than their scores in the process standards. While their mean score of the process standards was 2.292, their mean score of the content standards was 2.097. A two-tailed *t*-test showed that the mean scores between the process and content standards were statistically significant ($t = 1.973$, $p < 0.05$).

The data from another year ($n = 27$) were similar. The Spearman rank correlation was 0.867 ($p < 0.001$) and 72.1% of scores matched between the two raters. The candidates' mean score for all standards was 2.26. Their mean scores in content and process standards were 2.20 and 2.34 respectively. The candidates' scores were the highest in mathematical communication and mathematical connections (2.44 and 2.46 respectively). They scored lowest in discrete mathematics (2.09). On an individual basis, the highest average score of 13 standards was 3 and the lowest average score was 1.65. Most of the individual average scores clustered around 2.0. A total of 23 candidates successfully completed their portfolio in their first attempt, while the remaining 4 candidates completed the portfolio in their second attempt. The data in other years were similar.

There were some common findings in all five years. Interrater consistency was generally strong. Although the candidates found the portfolio assignment to be difficult, it allowed candidates to demonstrate their strengths by showcasing mathematical problems and entries they were passionate about. Average and weak candidates had difficulty connecting various mathematical topics such as linear and abstract algebra. They also found it difficult to connect discrete mathematics to other areas such as algebra and geometry. Even though one of the purposes of discrete mathematics is to introduce various methods of doing proofs, the majority of candidates were not able to connect discrete mathematics (a content standard) to proof and reasoning (a process standard).

The portfolio system allows teacher candidates to reflect on how mathematical knowledge they have learned at the university level is deeply connected to the secondary school mathematics they will be teaching in the future. The raters were able to differentiate a strong portfolio from a weak portfolio in a relatively short amount of time. There was a high degree of match between the two raters if candidates demonstrated high performance in their portfolios. The match was also high if candidates demonstrated low performance in their

portfolios. We have used the portfolio data for our NCATE/NCTM accreditation and have received positive feedback about the collaboration between the Mathematics Department and the Education Department.

Based on our data analysis, we have found portfolio assessment to be valid and reliable. It measures what we intended to measure, teacher candidates' mathematical knowledge of content and process standards as described in the NCATE/NCTM standards. The portfolio assessment was also found to be reliable as indicated by the Spearman rank correlation and the percent matching of scores between the two raters.

IMPLICATIONS OF THE STUDY: POSSIBLE PROGRAM IMPROVEMENTS

An obvious goal for a mathematics major pursuing secondary certification is that he or she has a deep understanding of mathematics and is able to make connections between that mathematical knowledge and the topics he or she will teach. The *Advanced Mathematics for High School Teaching* course is the ideal setting to investigate the degree to which this actually occurs at our university. The data from student portfolios have been invaluable to both departments. For example, we have found that the portfolio assessment should emphasize the use of fewer problems that have rich connections, rather than more problems at a superficial level.

We have increased the communication about student performance with other colleagues in the Mathematics Department. In the future we will ensure that candidates have met all prerequisites before enrolling in *Advanced Mathematics for High School Teaching*. In order to improve potential teacher candidates' knowledge of mathematical contents and processes, the candidates need a better opportunity to learn discrete math topics like graph theory and mathematical induction. The Mathematics Department is examining ways to include this content within its mathematics major courses. In addition, the content of *Advanced Mathematics for High School Teaching* has been revised to include topics in which students struggled in their portfolios in part because the topics appear to be missing or under emphasized in the mathematics major courses.

Our project is based both on research and practice. The project has helped us to identify what mathematical content knowledge the teacher candidates know or do not know and address the issues. Although this paper is focused on one institution's effort to improve teachers' content knowledge of mathematics required for teaching in secondary school, other institutions can benefit from this project. We are happy to share our research and materials with anyone who is interested.

LIMITATIONS OF THE STUDY

Our study has some limitations. Although we are confident that the portfolio entries demonstrate candidates' understanding of mathematical content and process standards as described in the NCATE/NCTM standards, it is hard to classify the entries based on Mathematical Knowledge for Teaching (MKT) as described by Hill and colleagues. Most of the portfolio entries submitted by candidates can be classified as common content knowledge (CCK). A few can be classified as specialized content knowledge (SCK). This portfolio does

not focus on knowledge of content and students (KCS) and knowledge of content and teaching (KCT). Our candidates demonstrate how their mathematical knowledge is connected to the teaching of secondary mathematics without getting into the classification of MKT.

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Appendix A: Portfolio Rubric

Target (Score 3: Evidence of exceptional performance, beyond what one would typically expect of a mathematics major.)

The student demonstrates: a) in-depth knowledge and comprehensive understanding of the mathematics content and processes from the NCATE/NCTM standards and indicators, with no significant mathematical errors, b) an ability to reflect deeply on and communicate clearly, logically, and elegantly about the mathematics included in the standards and indicators, c) an inherent connection among multiple mathematical content areas and processes, d) an excellent knowledge of the historical development of the content (number and number systems, algebra, Euclidean and non-Euclidean geometries, calculus, discrete mathematics, statistics and probability, measurement and measurement systems) including contributions from diverse cultures, and e) how college level mathematics is connected to the teaching of high school mathematics.

Acceptable (Score 2: Evidence of satisfactory performance and above.)

The student demonstrates: a) a solid knowledge and understanding of the mathematics content and processes from the NCATE/NCTM standards and indicators, although some mathematical errors may occur, b) an ability to reflect on and communicate clearly and logically about the mathematics included in the standards and indicators, c) some connections among multiple mathematical content areas and processes, d) a knowledge of the historical development of the content (number and number systems, algebra, Euclidean and non-Euclidean geometries, calculus, discrete mathematics, statistics and probability, measurement and measurement systems) including contributions from diverse cultures, and e) some connections between college and high school mathematics.

Unacceptable (Score 1: Unsatisfactory performance.)

The student demonstrates at least one of the following: a) a lack of knowledge or understanding of the mathematics content and processes from the NCATE/NCTM standards and indicators or substantial mathematical errors related to the standards and indicators, b) a lack of ability to reflect on or communicate logically about the mathematics included in the standards and indicators, c) no connections between two or more mathematical content areas and processes, d) a lack of knowledge of the historical development of the content including contributions from diverse cultures, or e) no connections between college and high school mathematics.

Missing or Difficult to Make Sense (Score 0: Unsatisfactory)

The student does not provide any response or the response provided is extremely hard to interpret or make sense.